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Multiaxial high cycle fatigue criteria applied to motor crankshafts

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Abstract

A comparative study is made of the applicability of critical plane based multiaxial high cycle fatigue models to predicting the fatigue behavior of metallic materials. A number of models, namely Matake, McDiarmid, Carpinteri and Spagnoli, Liu and Mahadevan and Papadopoulos, were applied to fatigue limit states, involving synchronous fully reversed in-phase sinusoidal bend and torsion loading. The results obtained indicated a good predictive capability of the models with an average error index situated approximately between -5,5% and 4,5%. However, this average was limited to less than 3% for the latter three models. Finally, the critical plane orientation, which, for a given material, is characteristic of the proper model, is compared with that of the fracture plane, exclusively determined by the ratio between the shear stress and normal stress amplitudes.

Keywords: fully reversed loading; proportional loading; critical plane; fracture plane; error index.

1. Introduction

Fatigue is the most usual mechanical damage in all engineering fields. To prevent such a problem, a projection of the fatigue life incorporating numerical, analytical and experimental tests seem to be necessary. As many mechanical components, such as railroad wheels, crankshafts, axles and turbine blades, are expected to suffer multiaxial loading during service operations, the fatigue problem becomes more complex due to the complexity of the stress states, loading histories and different orientations of the initial crack. Accordingly, the need has been arising to introduce criteria capable of generalizing the fatigue limit concept so as to include multiaxial loading conditions. Such criteria can be divided into three groups: stress based, strain based and energy based models can be divided into three groups: stress based, strain based and energy based models.

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As the stress levels involved in high cycle fatigue are kept below the elastic limit, only a number of stress based models, namely Ma (Matake, 1977), Mc (McDiarmid, 1987; McDiarmid, 1991), F (Findley, 1959), C&S (Carpinteri and Spagnoli, 2001; Carpinteri et al., 2011; Carpinteri et al., 2013), L&M (Liu and Mahadevan, 2005) and P (Papadopoulus et al., 1997; Papadopoulos, 2001) are considered in the present work.

The main purpose here is to test the applicability of the aforementioned models to some experimental loading conditions, available in the literature (Nishihara & Kawamoto, 1945; Zenner et al., 1985), involving synchronous fully reversed sinusoidal in-phase bend and torsion loading applied to a variety of metallic materials with different fatigue behaviors. It is to be emphasized that all these loading conditions correspond to the fatigue limit state that represents the multiaxial stress field above which fracture occurs and below which no fracture will take place, in analogy with the fatigue limit stress for uniaxial loading.

Whereas the Papadopoulos criterion can be directly applied by knowing the applied stress amplitudes, the other models depend for their application on the prior identification of the critical plane, where fatigue damage can occur leading to crack nucleation. Once the orientation of this plane is identified, the normal and shear stress amplitudes, acting on it, can be determined and fatigue failure assessment can thus be presented in the form of inequality. The relative difference between the two sides of the inequality is referred to as the error index, and, for a given fatigue limit state, it can be null, positive or negative. The comparison of the error index associated with the application of the models in question is therefor expected to provide a good assessment of their predictive capabilities in defining the fatigue behavior.

2. Multiaxial high cycle fatigue criteria

The inequalities representative of the Matake, McDiarmid, Findley, Carpinteri and Spangnoli, Liu and Mahadevan and Papadopoulos criteria are given, respectively, by expressions (1) to (6):

$$C_a + \mu\, N_{max} \ \leq t_{-1} \tag{1}$$

$$C_a + \frac{t_{-1}}{2\sigma_u} N_{max} \ \leq t_{-1} \tag{2}$$

$$C_a + k\, N_{max} \ \leq f \tag{3}$$

$$\sqrt{N_{max}^2 + (\frac{f_{-1}}{t_{-1}})^2\, C_a^2} \ \leq f_{-1} \tag{4}$$

$$\sqrt{[\frac{N_a + \left(1 + \eta \frac{N_m}{f_{-1}}\right)}{f_{-1}}]^2 + (\frac{C_a}{t_{-1}})^2} \ \leq \lambda \tag{5}$$

$$\sqrt{\frac{\sigma_a^2}{3} + \tau_a^2} + \alpha \frac{\sigma_a + \sigma_m}{3} \ \leq t_{-1} \tag{6}$$

C_a and N_{max}, in the expressions above, are, respectively, the shear stress amplitude and the maximum normal stress acting on the critical plane. N_{max} is given by:

$$N_{max} = N_a + N_m \tag{7}$$

where N_a is the amplitude and N_m the mean value.

The constants μ, k, f, η, λ and α are material parameters, which depend exclusively, as shown in Table 1, on the fatigue limits for fully reversed bending f_{-1} and fully reversed torsion t_{-1}. The constant δ in this table is presented later as a function of s, where s is the ratio between t_{-1} and f_{-1}. Applying the McDiarmid criterion, one needs to know, in the addition to t_{-1}, the ultimate tensile strength σ_u.

Different from the critical plane approach, the Papadopoulos criterion which is based on the mesoscopic scale approach is applied by simply substituting the applied normal stress and shear stress amplitudes σ_a and τ_a, together with the mean stress σ_m in expression (6).

For fully reversed loading, which is the type of loading considered in the present work, σ_m and N_m are taken to be null and hence N_{max} is to be replaced by N_a in the inequalities (1) to (4).

Table 1. Definition of the pertinent material constants.

Material constants	Definitions	Material constants	Definitions
μ	$2\,(t_{-1}/f_{-1})-1$	η	$3/4+\ 1/4[(\sqrt{3}-\ f_{-1}/t_{-1})/(\sqrt{3}-1)]$
k	$[2-(f_{-1}/t_{-1})]/[\ 2\sqrt{(f_{-1}/t_{-1})-1}]$	λ	$[cos^2(2\delta)s^2+sin^2(2\delta)]^{1/2}$
f	$\sqrt{f_{-1}^2/4[(f_{-1}/t_{-1})-1]}$	α	$[t_{-1}-(f_{-1}/\sqrt{3})]/(f_{-1}/3)$

3. Critical plane identification

This can be achieved by first considering a general material plane oriented at an angle ψ (Fig.1), where the stress amplitudes C_a and N_a acting on such a plane due to applied synchronous sinusoidal bending and torsion are given by (Papadopoulus et al., 1997):

$$C_a = \sqrt{(-\frac{\sigma_a}{2} sin2\psi + \tau_a\, cos2\psi\, cos\beta)^2 + {\tau_a}^2\, cos^2 2\psi\, sin^2\beta} \tag{8}$$

$$N_a = |cos\psi|\sqrt{{\sigma_a}^2 cos^2\psi + 4{\tau_a}^2\, sin^2\psi + 2\sigma_a\tau_a\, sin2\psi\, cos\beta} \tag{9}$$

where β is the phase difference between shear and normal stresses.

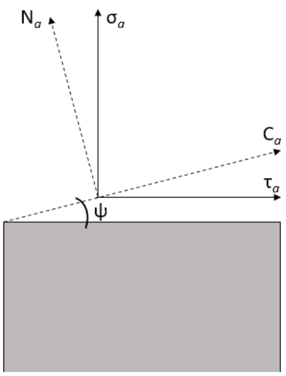

Fig. 1. Schematic representation of normal and shear stress amplitudes acting on an arbitrary plane defined by the angle ψ.

For in-phase bend and torsion loadings, the angle β is taken to be nil and the expressions for C_a and N_a are significantly simplified.

Both the Matake and McDiarmid models refer to the critical plane as the plane on which the shear stress amplitude C_a reaches its maximum. Accordingly the angle ψ_c that defines the critical plane orientation can be determined by enumeration, where the angle ψ, in equation (8), is varied by 0.1° increment within the range from 0° to 360°. In the event of having two or more angles corresponding to the C_a maximum, the angle among them, which is associated with the highest N_a level, should be taken as ψ_c. Knowing ψ_c, the corresponding N_a and C_a values can be calculated and then substituted in the left hand side (LHS) of inequalities (1) and (2).

In regard to the Findley model, the critical plane orientation is defined by maximizing the linear combination $(C_a + kN_a)$ with respect to ψ. Again with ψ_c already known, N_a and C_a can be calculated and the LHS of inequality (3) can be determined.

Identification of the critical plane for both C&S and L&M models depend, in the first place, on determining the fracture plane orientation. For a given loading history, the fatigue fracture plane is identified as the material plane normal to the maximum principal stress. Accordingly, the angle ψ_f that defines the fracture plane orientation can be determined by maximizing N_a (9), with respect to ψ. Knowing ψ_f, ψ_c is given by (Carpinteri and Spagnoli, 2001; Liu and Mahadevan, 2005):

$$\psi_c = \psi_f + \delta \tag{10}$$

where δ is the angle between the fracture plane and the critical plane, and is given by:

$$\delta = \left[1 - \left(\frac{t_{-1}}{f_{-1}}\right)^2\right]\frac{3\pi}{8} \tag{11}$$

for the C&S model (Carpinteri and Spagnoli, 2001), and by:

$$\delta = \frac{1}{2}cos^{-1}\left[\frac{-2+\sqrt{4-4(\frac{1}{s^2}-1)(5-\frac{1}{s^2}-4s^2)}}{2(5-\frac{1}{s^2}-4s^2)}\right] \tag{12}$$

for the L&M model (Liu and Mahadevan, 2005).

4. Results and discussion

The fracture plane orientation ψ_f, which is determined by the applied stress amplitudes, is in fact unique for all the models. More specifically, the higher the ratio σ_a/τ_a, the lower the angle ψ_f, consistent with the fact that ψ_f tends to zero for uniaxial normal stress and to 45° for pure shear loading. On the other hand, the critical plane orientation, ψ_c, corresponding to a given loading condition, varies appreciably from one model to another. For a given model, ψ_c depends on the applied stress amplitudes as well as on the fatigue properties of the material.

The error index, *I*, associated with the application of any of the six models, refers to the relative difference between the two sides of the inequality. *I* can thus be expressed as:

$$I = \frac{LHS-RHS}{RHS} \times 100 \tag{13}$$

The values of *I* corresponding to the different loading conditions are listed in Table 2, for the variety of materials involved. Except for a few cases, the vast majority of the *I* values are situated within the range -10% to 10%, indicating a good predictive capability of the criteria in question. This is also demonstrated by Fig. 2, where the overall average values of *I* are also shown. One can thus conclude that, except for the McDiarmid model, the others are moderately conservative, with the C&S, L&M and P models exhibiting the lowest error compared to other three.

Table 2. Error index, corresponding to the multiaxial fatigue criteria, for the different loading conditions.

		Index Error - *I* (%)					
σ_a(MPa)	τ_a(MPa)	Ma	Mc	F	C&S	L&M	P
Material: Hard steel: f_{-1} = 313.9 MPa; t_{-1} = 196.2 MPa; σ_u = 704.1 MPa							
327.7	0.0	4.4	-4.8	4.4	1.8	4.3	4.4
308.0	63.9	4.6	-4.1	4.6	1.2	3.3	3.8
255.1	127.5	8.2	1.0	8.2	3.1	5.3	5.5
141.9	171.3	3.6	-0.4	3.5	-1.5	0.3	0.2
0.0	201.1	2.5	2.5	2.5	1.9	2.3	2.5
Material: Hard steel: f_{-1} = 313.9 MPa; t_{-1} = 196.2 MPa; σ_u = 680.0 MPa							
138.1	167.1	1.0	-2.8	0.9	-3.9	-2.8	-2.3
245.3	122.65	4.0	-2.6	4.0	-0.8	1.3	1.4
299.1	62.8	1.7	-6.3	1.7	-1.5	0.3	0.9
Material: 42CrMo4: f_{-1} = 398.0 MPa; t_{-1} = 260.0 MPa; σ_u = 1025.0 MPa							
328.0	157.0	6.8	-4.7	6.7	0.7	4.5	4.2
233.0	224.0	10.9	2.8	10.8	4.1	6.6	7.3
Material: 34Cr4: f_{-1} = 410.0 MPa; t_{-1} = 256.0 MPa; σ_u = 795.0 MPa							
314.0	157.0	2.0	-3.4	2.0	-2.8	-0.8	-0.5
Material: 30NCD16: f_{-1} = 660.0 MPa; t_{-1} = 410.0 MPa; σ_u = 1880.0 MPa							
485.0	280.0	4.7	-3.2	4.7	-0.3	1.1	1.8
Material: Mild steel: f_{-1} = 235.4 MPa; t_{-1} = 137.3 MPa; σ_u = 518.8 MPa							
245.3	0.0	4.2	1.1	4.2	3.9	4.0	4.1
235.6	48.9	7.2	4.3	7.2	5.9	6.0	6.3
187.3	93.6	7.8	5.5	7.8	4.7	5.0	5.0
101.3	122.3	2.6	1.3	2.5	-1.0	-1.0	-0.8
0.0	142.3	3.6	3.6	3.6	3.6	4.0	3.6
Material: Cast iron: f_{-1} = 96,1 MPa; t_{-1} = 91,2 MPa; σ_u = 230.0 MPa							
93.2	0.0	-3.0	-38.7	-3.0	-3.6	-13.9	-3.0
95.2	19.7	3.4	-33.1	3.4	2.5	-26.3	2.8
83.4	41.6	5.6	-26.3	5.6	4.0	4.8	3.8
56.3	68.0	8.5	-13.2	8.4	5.4	6.8	5.9
0.0	94.2	3.3	3.3	3.3	-1.7	2.7	3.3

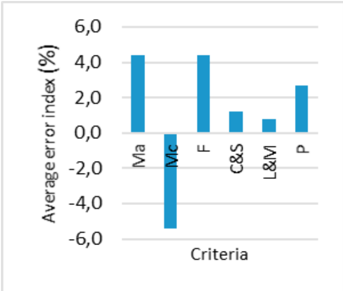

Fig. 2. Comparative presentation of the average value of the error index, for the fatigue criteria in question.

5. Conclusions

Based on what is presented above, the following conclusions can be drawn:

- Fracture plane orientation, in fully reversed multiaxial fatigue loading, is exclusively determined by the ratio between the shear and normal stress amplitudes. On the other hand, for a given loading condition, the critical plane orientation depends on the adopted multiaxial fatigue criterion. Whereas the Matake and McDiarmid models possess the same critical plane, the C&S and L&M models indicate critical planes with orientations that are close to each other.
- Critical plane orientation predicted by the Findley criterion is generally close to that the defined by the Matake model.
- The overall average of the error index I is limited to -5,5% $\leq I \leq$ 4,5%, indicating reasonable predictive capability of the models in question in defining fatigue behavior.
- Except for the McDiarmid criterion, the models are seen to be conservative as they mostly exhibit positive I values.

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